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A STUDY OF THE EFFECTS OF DISK FLEXIBILITY ON THE ROTORDYNAMICS OF THE SPACE SHUTTLE MAIN ENGINE TURBO-PUMPS

Prepared by:	George T. Flowers
Academic Rank:	Assistant Professor
University and Department:	University of South Florida Department of Mechanical Engineering
NASA/MSFC:	
Laboratory:	Structures and Dynamics
Division:	Control Systems
Branch:	Mechanical Systems Control
MSFC Colleague:	Donald P. Vallely
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ABSTRACT

Rotordynamical analyses are typically performed using rigid disk models. Studies of rotor models in which the effects of disk flexibility have been included indicate that it may be an important effect for many systems. This work addresses this issue with respect to the Space Shuttle Main Engine high pressure turbo-pumps. Finite element analyses have been performed for a simplified free–free flexible disk rotor model and the modes and frequencies compared to those of a rigid disk model. The simple model was then extended to a more sophisticated HPTOP rotor model and similar results were observed. Equations have been developed that are suitable for modifying the current rotordynamical analysis program to account for disk flexibility. Some conclusions are drawn from the results of this work as to the importance of disk flexibility on the HPTOP rotordynamics and some recommendations are given for follow–up research in this area.

ACKNOWLEDGEMENTS

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NOMENCLATURE

h = disk thickness coordinate
$I_{a,i}$ = axial moment of inertia
$I_{t,i}$ = transverse moment of inertia
$I_{k,i} = \int\int\int (h^2 r\rho) dr d\nu dh$
M = mass of rotor
m = mass matrix of rotor
q_i = i^{th} generalized coordinate
r = disk radial coordinate
x_i, y_i, z_i = Translational coordinates of rotor disk hub
X = Lateral translational coordinate of rotor disk hubs
ξ_i = i^{th} generalized coordinate for non – rotating system
Φ = mode shape
Ψ = mode shape
Δ = mode shape
ν = disk angular coordinate
ρ = material density
$\Gamma = \{\Psi\}^T\{I_b\}\{\Psi\}$
$\Lambda = \{\Psi\}^T\{I_h\}\{\Psi\}$
Ω = Rotor speed
θ_y, θ_z, θ_x = Euler angles

INTRODUCTION

In the modeling of any physical device or process, certain assumptions and restrictions must be made. It is important to carefully assess their validity in order to determine the accuracy and range of applicability of the mathematical model. The current procedure for analyzing the rotordynamics of the Space Shuttle Main Engine Turbo-pumps consists of coupling the free–free rotor and housing modes with constraints to produce a model for the complete turbo–pump. This model is then used in stability analyses and for time response simulations.

The standard practice in obtaining the free–free rotor modes is to neglect the effects of rotor disk flexibility. Research into this area has indicated that disk flexibility may play an important role in the rotordynamical behavior of turbo–machinery. If rotor disk flexibility has a significant effect on the rotordynamics of the SSME turbo–pumps, significant errors could be introduced into analyses. So, it is important to assess such effects and develop means of accounting for it in analysis procedures. This work attempts to address this issue by examining some simplified finite element rotor motors and developing analytical methods of dealing with disk flexibility effects.

OBJECTIVES

(1.) Develop finite element rotor models to evaluate the influence of disk flexibility on the rotor free–free modes.

(2.) If justified by (1.), develop methods of modifying the current rotordynamical analysis program to account for rotor disk flexibility.

3.) Develop recommendations for follow–up research.

PREVIOUS RESEARCH

There is a large body of research that is available on the dynamical behavior of rotating flexible bodies. A classic work by Lamb and Southwell presented a discussion of the vibrational behavior of a spinning disk.[1] This work served as the basis for much further work in this area. Likins performed a study of mathematical modeling of spinning elastic bodies for use in modal analyses.[2] Wilgens and Schlack investigated the dynamical behavior of a flexible beam attached to a rotating shaft.[3] Brown and Schlack extended this work to a study of the stability of a spinning body.[4] Pringle presented a method for examining the dynamical behavior of a system with connected moving parts.[5] Laurenson discussed methods for performing modal analysis on rotating flexible structures.[6] Meirovitch presented an analytical study of discretization methods for flexible gyroscopic systems and drew conclusions concerning the appropriateness of various techniques.[7]

There is also a fairly large body of work documented in the literature concerning studies of disk flexibility on rotors and turbomachinery. Vance studied several rotor systems, performing a combined experimental – analytical investigation.[8] His analytical model was configured so that the effects of disk flexibility could be accounted for in an approximate fashion. These studies indicated that, for most cases, the inclusion of rotor disk flexibility could significantly improve the correlation between experimentally measured rotor free–free natural frequencies and calculated values. Klompas developed a technique for studying shaft whirling which included the effects of flexible disks and blades.[9] Klompas extended his study further by including the effects of disk flexibility in a study of the unbalance response of a turbo–machine.[10] This work was primarily aimed at a study of the effects of blade loss. Palladine and Rosettos developed a finite element method for examining the effects of flexibility on the behavior of a rotor.[11] Wilgen and Schlack investigated the effects of disk flexibility on shaft whirl stability using Liapunov techniques.[12] The resulting procedure is analytically very nice, but for complicated disk shapes and multi–disk systems the method could quickly become intractable. Dopkin and Shoup performed a study of the effects of disk flexibility on the resonant frequencies of an axisymetric rotating shaft. They found that the effects of disk flexibility may significantly reduce the rotor resonant speeds and that this effect was particularly pronounced at low rotor speeds.[13] Shahab and Thomas studied finite element models of single and multi–disk rotor systems and compared the results to experimental models.[14] This study indicated that coupling effects between the shaft and disk modes can have a significant effect on the dynamical behavior of a rotor. Sakata, Aiba, and Ohnabe studied the transient vibration behavior of a rotor subjected to a blade loss and included the effects of disk flexibility.[15]

FINITE ELEMENT ANALYSES

The first objective of this research effort is aimed at evaluating the influence of disk flexibility on the rotor free-free modes. The most straight–forward way of studying this effect is by developing finite element rigid and flexible disk rotor models and comparing the rotor free–free modes.

Examination of schematics of the HTOTP and HTFTP rotor disk configurations reveals that the second turbine stage of the HTOTP has the thinnest disk configuration and is probably most likely to exhibit flexibility effects. In order to establish what types of behavior might be expected for a rotor with a flexible disk, a simple model was first examined. A configuration was selected that is approximately that of the HTOTP rotor shaft with a disk attached to represent the second turbine stage. This model consists of a 0.051 m. diameter, 0.59 m. steel shaft. A 0.239 m. diameter, 0.0161 m. thick disk was attached to the shaft with its center point 0.528 m. from the end of the shaft. For the flexible disk study, the properties of steel were used for the disk. For the rigid disk study, the density and poisson's ratio of steel and a modulus of rigidity and modulus of elasticity three orders of magnitude above those of steel were used as the material properties. The free–natural frequencies for the two models are compared in Table 1 and the mode shapes are illustrated in Figure 1.

Table 1 : Free–Free Natural Frequencies
For Simple Rotor Models

Natural Frequencies (Hz)		
Mode	Flexible Disk	Rigid Disk
First Bending	487	502
Second Bending	1003.0, 1997.0	1225.0
Third Bending	3393.0	2453.0

Note that the first rotor bending mode is not significantly affected by the disk flexibility. However, the second and third bending modes are strongly influenced.

In order to better evaluate the influence of disk flexibility, it was decided to develop a second, more sophisticated rotor model of the HPTOP. A finite element code was developed based on work by Muller[10]. The model was developed using

ANSYS, a finite element analysis package. The types of beams and rigid masses used are the same as those used by Muller except the second turbine has been replaced by a 0.239 m. diameter, 0.0161 m. thick disk, and rigid inertias $I_{xx} = 5.28x10^{-3}$ kg – m^2, $I_{yy} = 2.96x10^{-3}$ kg – m^2, and $I_{zz} = 2.96x10^{-3}$kg – m^2. As in the previous study, the material properties of steel are used for the flexible disk case. For the rigid disk, the density and poisson's ratio of steel are used with a modulus of elasticity and modulus of rigidity increased by three orders of magnitude. The resulting model was examined using an eigenanalysis to determine the free–free rotor modes.

Table 2 : Free–Free Natural Frequencies For HPOTP Rotor Models

Natural Frequencies (Hz)		
Mode	Flexible Disk	Rigid Disk
First Bending	462.2	467.6
First Torsional	913.8	944.3
Second Bending	926.7, 1652.0	1072.0
Second Torsional	1255.2	1274.0
Third Bending	1773.6	1983.0
Third Torsional	2636.7	2486.0

For the first bending modes, the mode shapes and frequencies for the two cases match closely. Similar behavior is observed for the first torsional modes. For the second bending modes, the rigid disk system has a single mode shape. The flexible disk model exhibits two second bending modes. The first is characterized by motion of the disk in–phase with the hub. The second is characterized by motion of the disk out–of–phase with the hub. The in–phase motion serves to effectively reduce the second bending mode natural frequency and the out–of–phase motion serves to effectively increase it.

ANALYTICAL STUDY

The results of the finite element studies indicate that rotor disk flexibility can significantly alter the free–free rotor modes and frequencies. As a result, it is appropriate to develop techniques for accounting for such effects in the current rotordynamical analysis program so that the significance of disk flexibility can be evaluated for the complete turbo–pump model.

The equations of motion for two formulations of a flexible rotor – flexible disk model are presented. Each of the approaches presented in this study assume that the deflection of the rotor disk is primarily in the lateral direction. The beam is assumed to be axially and torsionally rigid, which implies that the disk hubs move together as a rigid body in the lateral direction. In addition, it is assumed that the beam is axially stiff so that the axial hub position of ith disk is $x_i \equiv X$, for all i. For each of the following developments, the rotor is considered to consist of a series of flexible disks. The equations are formulated using a Lagrangian formulation.

Rotation Sequence

$\theta_{y,i}$ about y
$\theta_{z,i}$ about z'
$\theta_{x,i}$ about $\bar{x}$
ν about $\bar{i}$ (disk angular coordinate)

Formulation Using Flexible Rotor – Flexible Disk Modes

Position of Arbitrary Point In Inertial Coordinate System

First, obtain the position vector $\vec{P}_i$ for an arbitrary point on the i^{th} rotor disk.

$\vec{P}_i = P_{x,i}\hat{x} + P_{y,i}\hat{y} + P_{z,i}\hat{z}$

where

$P_{x,i} = u_i$

$$P_{y,i} = Y_i + h\sin(\theta_{z,i}) + r\cos(\nu + \theta_{x,i})\cos(\theta_{z,i})$$

$$P_{z,i} = Z_i + r\sin(\nu + \theta_{x,i})\cos(\theta_{y,i}) - h\cos(\theta_{z,i})\sin(\theta_{y,i})$$
$$+r\cos(\nu + \theta_{x,i})\sin(\theta_{y,i})\sin(\theta_{z,i})$$

Velocity of Arbitrary Point

Differentiation of the position vector with respect to time yields a velocity vector for the arbitrary point.

$$\vec{V_i} = V_{x,i}\hat{x} + V_{y,i}\hat{y} + V_{z,i}\hat{z}$$

where

$$V_{x,i} = \dot{u}_i$$

$$V_{y,i} = \dot{Y} - \dot{\theta}_{z,i}h\cos(\theta_{z,i}) - r\Omega\sin(\nu + \theta_{x,i})\cos(\theta_{z,i}) - r\dot{\theta}_{z,i}\cos(\nu + \theta_{x,i})\sin(\theta_{z,i})$$

$$V_{z,i} = \dot{Z}_i + r\Omega\cos(\nu + \theta_{x,i})\cos(\theta_{y,i}) - r\dot{\theta}_{y,i}\sin(\nu + \theta_{x,i})\sin(\theta_{y,i})$$
$$+h\dot{\theta}_{z,i}\sin(\theta_{z,i})\sin(\theta_{y,i}) - h\dot{\theta}_{y,i}\cos(\theta_{z,i})\cos(\theta_{y,i})$$
$$-r\Omega\sin(\nu + \theta_{x,i})\sin(\theta_{y,i})\sin(\theta_{z,i}) + r\dot{\theta}_{y,i}\cos(\nu + \theta_{x,i})\cos(\theta_{y,i})\sin(\theta_{z,i})$$
$$+r\dot{\theta}_{z,i}\cos(\nu + \theta_{x,i})\sin(\theta_{y,i})\cos(\theta_{z,i})$$

Kinetic Energy

Express the kinetic energy of each disk as

$$T_i = \frac{1}{2}\int\int\int(V_{x,i}{}^2 + V_{y,i}{}^2 + V_{z,i}{}^2)\rho r dr d\nu dh$$

The total kinetic energy of the rotor is then

$$T = \Sigma_i\, T_i$$

$$T = \frac{1}{2}\int\int\int(\{\dot{u}\}^T\{\dot{u}\})r dr d\nu dh$$

$$+\frac{1}{2}\{\dot{\theta}_z\}^T[\mathrm{I_h}]\{\dot{\theta}_z\} +\frac{1}{2}\{\dot{\theta}_y\}^T[\mathrm{I_h}]\{\dot{\theta}_y\} +\frac{1}{2}\Omega\{\dot{\theta}_y\}^T[\mathrm{I_a}]\{\theta_z\} +\frac{1}{2}\Omega\{\dot{\theta}_z\}^T[\mathrm{I_a}]\{\theta_y\}$$
$$+\frac{1}{2}\Omega^2[\mathrm{I_a}] +\frac{1}{2}\{\dot{\mathrm{Y}}\}^T[\mathrm{m}]\{\dot{Y}\}^2 +\frac{1}{2}\{\dot{\mathrm{Z}}\}^T[\mathrm{m}]\{\dot{Z}\}^2 -\frac{1}{4}\Omega^2\{\theta_z\}^T[\mathrm{I_a}]\{\theta_z\}$$
$$-\frac{1}{4}\Omega^2\{\theta_y\}^T[\mathrm{I_a}]\{\theta_y\}$$

where $I_{h,i} = \int\int\int(h^2\rho r)drd\nu dh$

$$\{I_h\} = \begin{pmatrix} I_{h,1} & 0 & 0 & . & . & 0 \\ 0 & I_{h,2} & 0 & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & I_{h,n} \end{pmatrix}$$

$$\{u\} \equiv \begin{pmatrix} u_1 \\ u_2 \\ . \\ . \\ . \\ u_n \end{pmatrix}$$

For a non–spinning rotor, the position of a point on the disk can be expressed in terms of the free–free rotor disk modes.

$$\{u\} = \{h\} + \{X\} + \begin{pmatrix} \Delta_z & 0 \\ 0 & \Delta_y \end{pmatrix} \begin{pmatrix} \xi_y \\ \xi_z \end{pmatrix}$$

where X is the hub axial position.

For a spinning rotor, it is necessary to transform this relation to account for the rotor spin. Notice that u_i is defined in terms of the inertial reference frame. In order to express u_i in terms of the free–free disk modes, one can make the following coordinate transformation.

$$\{\xi_y\} = \cos(\Omega t)\{q_y\} + \sin(\Omega t)\{q_z\}$$

$$\{\xi_z\} = \cos(\Omega t)\{q_z\} - \sin(\Omega t)\{q_y\}$$

$$\{u\} = \{h\} + \{X\} + \begin{pmatrix} \Delta_z & 0 \\ 0 & \Delta_y \end{pmatrix} \begin{pmatrix} \cos(\Omega t) & \sin(\Omega t) \\ -\sin(\Omega t) & \cos(\Omega t) \end{pmatrix} \begin{pmatrix} q_y \\ q_z \end{pmatrix}$$

Notice that since the rotor is axisymmetric,

$$\int\int\int([\Delta_y]^T[\Delta_y]r\rho)drd\nu dh = \int\int\int([\Delta_z]^T[\Delta_z]r\rho)drd\nu dh$$

Let $[A] = \int\int\int([\Delta_y]^T[\Delta_y]r\rho)drd\nu dh$

and

$$\begin{aligned} T = &\frac{1}{2}\{\dot{q}_y\}^T[\mathrm{A}]\{\dot{q}_y\} + \frac{1}{2}\{\dot{q}_z\}^T[\mathrm{A}]\{\dot{q}_z\} - \frac{1}{2}\Omega\{\dot{q}_z\}^T[\Gamma]\{q_y\} - \frac{1}{2}\Omega\{\dot{q}_y\}^T[\Gamma]\{q_z\} \\ &+\frac{1}{2}[\Omega][\mathrm{I_a}][\Omega] + \frac{1}{2}\mathrm{M}\dot{X}^2 + \frac{1}{2}\{\dot{q}_y\}^T\{\Phi\}^T\{m\}\{\Phi\}\{\dot{q}_y\} \\ &+\frac{1}{2}\{\dot{q}_z\}^T\{\Phi\}^T\{m\}\{\Phi\}\{\dot{q}_z\} - \frac{1}{4}\Omega^2\{q_z\}^T[\Gamma]\{q_z\} - \frac{1}{4}\Omega^2\{q_y\}^T[\Gamma]\{q_y\} \\ &+\frac{1}{2}\Omega\{\dot{\mathrm{q}}_\mathrm{y}\}^\mathrm{T}[\mathrm{A}]\{\mathrm{q_z}\} - \frac{1}{2}\Omega\{\dot{\mathrm{q}}_\mathrm{z}\}^\mathrm{T}[\mathrm{A}]\{\mathrm{q_y}\} + \frac{1}{2}\Omega\{\mathrm{q_z}\}^\mathrm{T}[\mathrm{A}]\{\dot{\mathrm{q}}_\mathrm{y}\} \\ &-\frac{1}{2}\Omega\{\mathrm{q_y}\}^\mathrm{T}[\mathrm{A}]\{\dot{\mathrm{q}}_\mathrm{z}\} + \frac{1}{2}\Omega^2\{q_y\}^T[\mathrm{A}]\{q_y\} + \frac{1}{2}\Omega^2\{q_z\}^T[\mathrm{A}]\{q_z\} \\ &+\frac{1}{2}\{\dot{q}_y\}^T[\mathrm{A}]\{\dot{q}_y\} + \frac{1}{2}\{\dot{q}_z\}^T[\mathrm{A}]\{\dot{q}_z\} \end{aligned}$$

$$\Gamma = \{\Psi\}^T[I_a]\{\Psi\}$$

$$\Lambda = \{\Psi\}^T[I_h]\{\Psi\}$$

Potential Energy

$$
[\omega_n^2] \equiv \begin{pmatrix} \omega_{n1}^2 & 0 & . & . & . & 0 \\ 0 & \omega_{n2}^2 & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & \omega_{n2}^2 \end{pmatrix}
$$

$$
[V] \equiv (\,\xi_y \quad \xi_z\,) \begin{pmatrix} [\omega_n^2] & 0 \\ 0 & [\omega_n^2] \end{pmatrix} \begin{pmatrix} \xi_y \\ \xi_z \end{pmatrix}
$$

$$
\equiv (\,q_y \quad q_z\,) \begin{pmatrix} \cos(\Omega t) & -\sin(\Omega t) \\ \sin(\Omega t) & \cos(\Omega t) \end{pmatrix} \begin{pmatrix} [\omega_n^2] & 0 \\ 0 & [\omega_n^2] \end{pmatrix} \begin{pmatrix} \cos(\Omega t) & \sin(\Omega t) \\ -\sin(\Omega t) & \cos(\Omega t) \end{pmatrix} \begin{pmatrix} q_y \\ q_z \end{pmatrix}
$$

$$
\equiv (\,q_y \quad q_z\,) \begin{pmatrix} [\omega_n^2] & 0 \\ 0 & [\omega_n^2] \end{pmatrix} \begin{pmatrix} q_y \\ q_z \end{pmatrix}
$$

where $[\omega_n^2]$ represents the squared natural frequencies of the flexible rotor – flexible disk.

Equations of Motion

$$
\{\Phi\}^T\{m\}\{\Phi\}\ddot{q_y} + [\mathrm{A}]\{\ddot{q_y}\} + [\Lambda]\{\ddot{q_y}\} + 2\Omega[\mathrm{A}]\{\dot{\mathrm{q}}_z\} + \frac{1}{2}\Omega^2[\Gamma]\{\mathrm{q}_y\} - \Omega^2[\mathrm{A}]\{q_y\} + [\omega_n^2]\{q_y\} = \{0\}
$$

$$
\{\Phi\}^T\{m\}\{\Phi\}\ddot{q_z} + [\mathrm{A}]\{\ddot{q_z}\} + [\Lambda]\{\ddot{q_z}\} - 2\Omega[\mathrm{A}]\{\dot{\mathrm{q}}_y\} + \frac{1}{2}\Omega^2[\Gamma]\{\mathrm{q}_z\} + \Omega^2[\mathrm{A}]\{q_z\} + [\omega_n^2]\{q_z\} = \{0\}
$$

$$
\{\ddot{X}\} = \{0\}
$$

CONCLUSIONS AND RECOMMENDATIONS

From the results of the finite element analyses, it is clear that rotor disk flexibility can significantly alter the rotor free–free modes and frequencies. While the first rotor bending mode is not strongly affected by disk flexibility, the second and third bending modes are significantly altered. In fact, two second bending modes are identified. The first is associated with in-phase motion of the disk with the rotor and the second is associated with out–of–phase motion of the disk with the rotor.

Equations have been developed for accounting for disk flexibility in a rotor model. Particular emphasis has been placed on obtaining equations that are suitable for incorporation into the current used rotordynamical analysis program.

From these results, the following conclusions and recommendations have been drawn.

1.) The rotordynamical analysis program should be modified to account for disk flexibility.
2.) The revised program should be tested with modal data from a simplified rotor finite element model.
3.) If results warrant, develop full–scale finite element models of the SSME turbo–pump rotors and use the resulting modal data in the revised rotordynamical analysis program.
4.) Compare these results to the responses predicted for a rigid disk rotor and evaluate the influence of rotor disk flexibility on the SSME turbo–pumps.
5.) In order to develop further physical insight into the effect of rotor disk flexibility, construct appropriately scaled rotor models and study their responses using a rotor test kit.
6.) Relate the results of these studies to actual observed behavior of the SSME turbo–pumps in order to gain physical insight and understanding of their rotordynamical behavior. Such understanding could enhance failure analysis .

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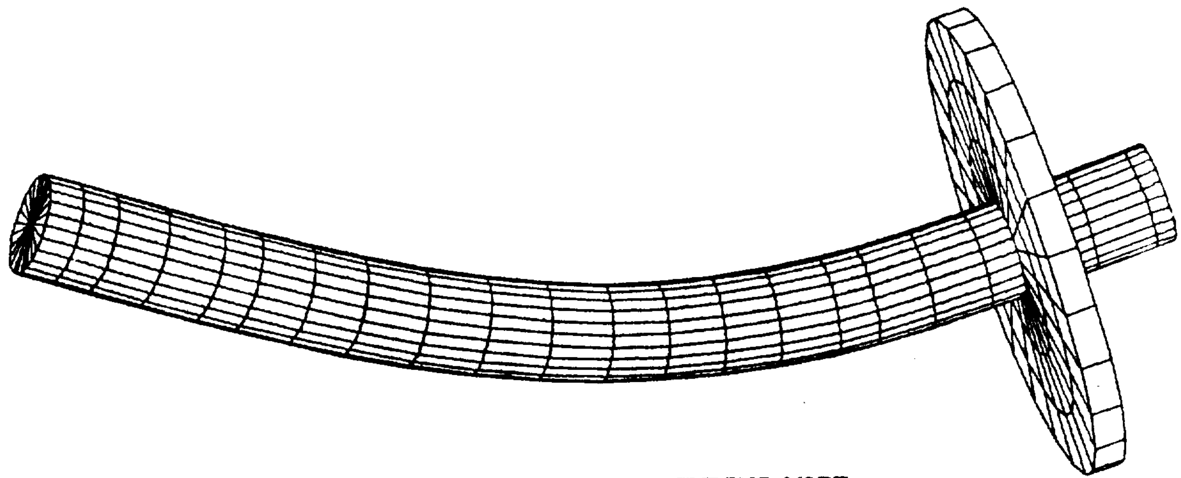

FIGURE 1.a: RIGID DISK MODEL - FIRST BENDING MODE
FREQUENCY = 502 Hz

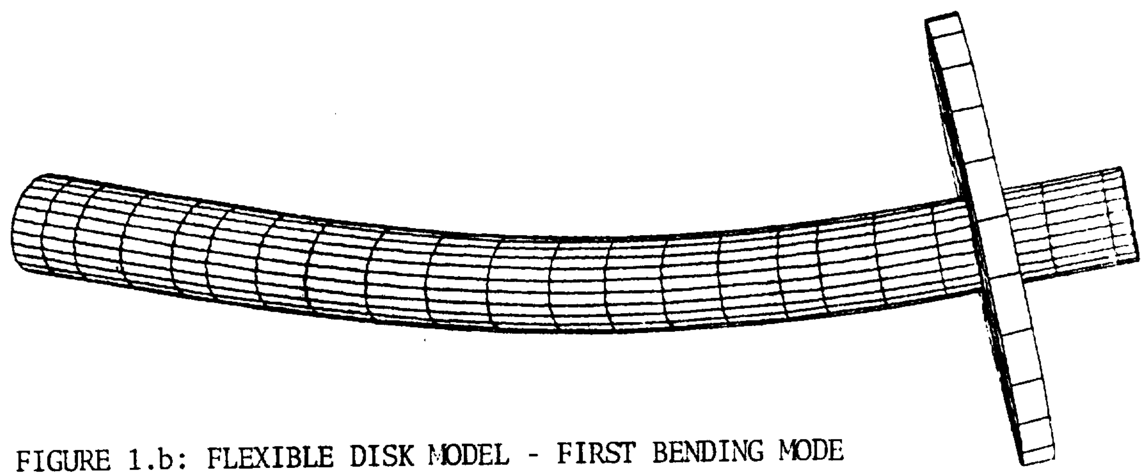

FIGURE 1.b: FLEXIBLE DISK MODEL - FIRST BENDING MODE
FREQUENCY = 487 Hz

FIGURE 1: SIMPLE ROTOR MODELS - FIRST BENDING MODES

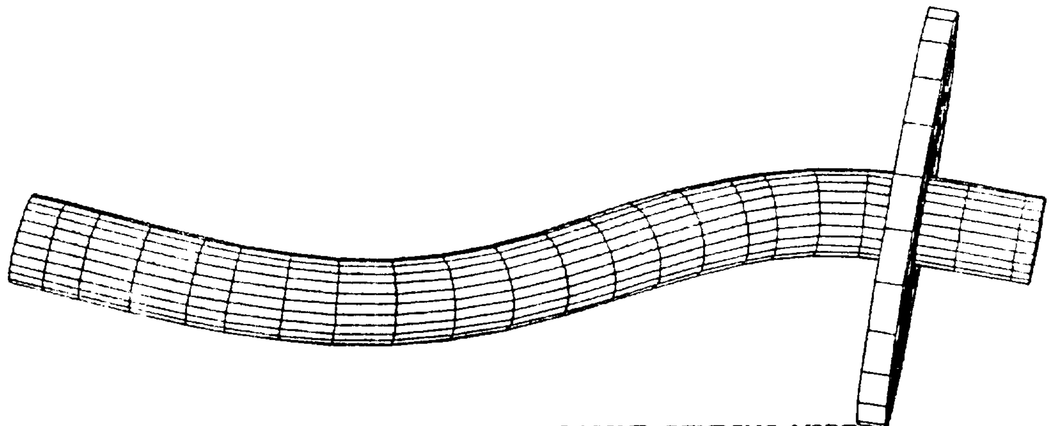

FIGURE 2.a: RIGID DISK MODEL - SECOND BENDING MODE
FREQUENCY = 1225 Hz

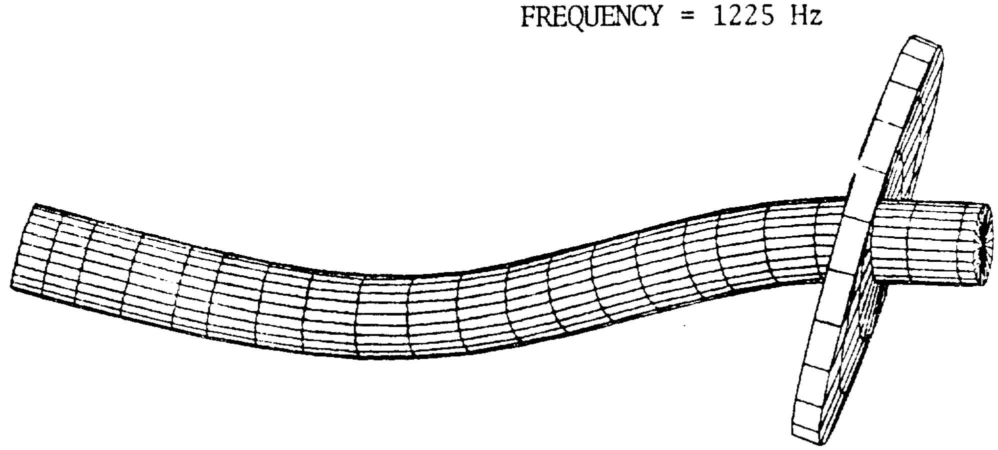

FIGURE 2.b: FLEXIBLE DISK MODEL - SECOND BENDING MODE, DISK IN-PHASE
FREQUENCY = 1003 Hz

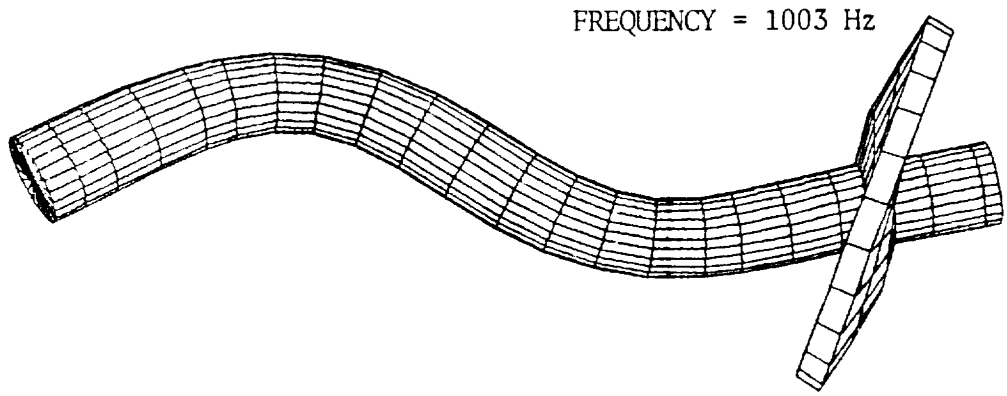

FIGURE 2.c: FLEXIBLE DISK MODEL - SECOND BENDING MODE, DISK OUT-OF-PHASE
FREQUENCY = 1997 Hz

FIGURE 2: SIMPLE ROTOR MODELS - SECOND BEARING MODES

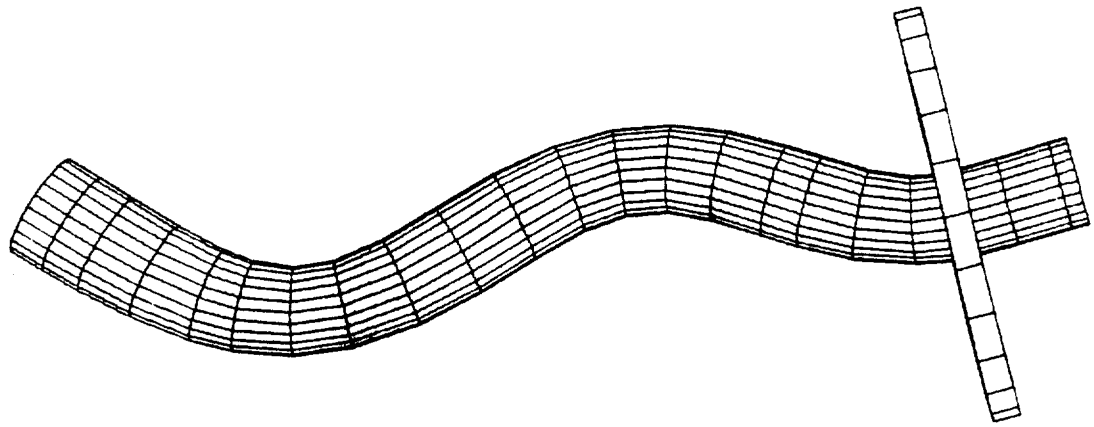

FIGURE 3.a: RIGID DISK MODEL - THIRD BENDING MODE
FREQUENCY = 2453 Hz

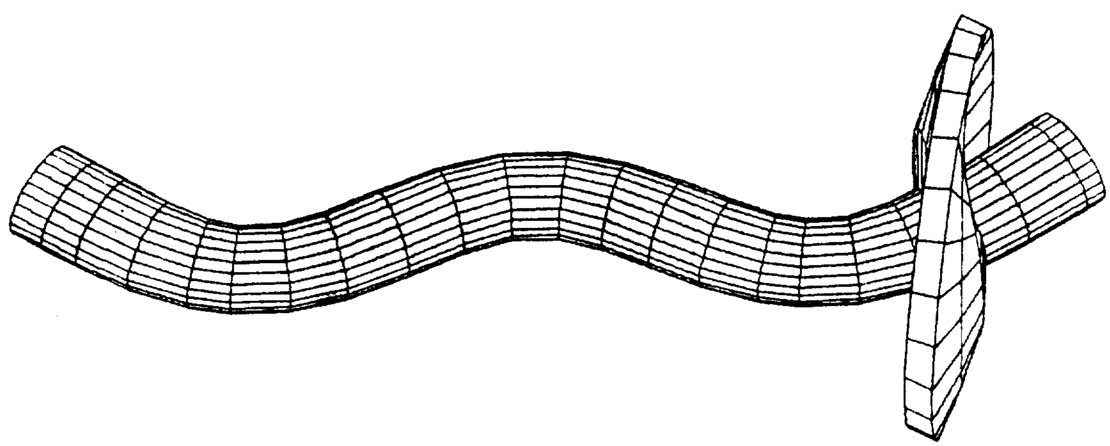

FIGURE 3.b: FLEXIBLE DISK MODEL - THIRD BENDING MODE, DISK OUT-OF-PHASE
FREQUENCY = 3393 Hz

FIGURE 3: SIMPLE ROTOR MODELS - THIRD BENDING MODES

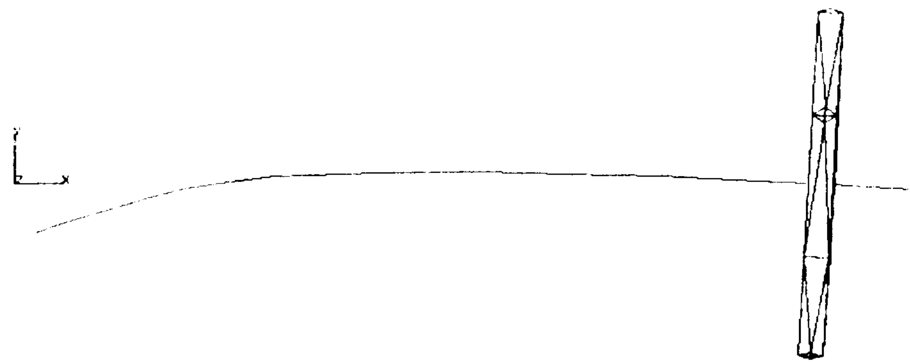

FIGURE 4.a: RIGID DISK MODEL - FIRST BENDING MODE
FREQUENCY = 467.6 Hz

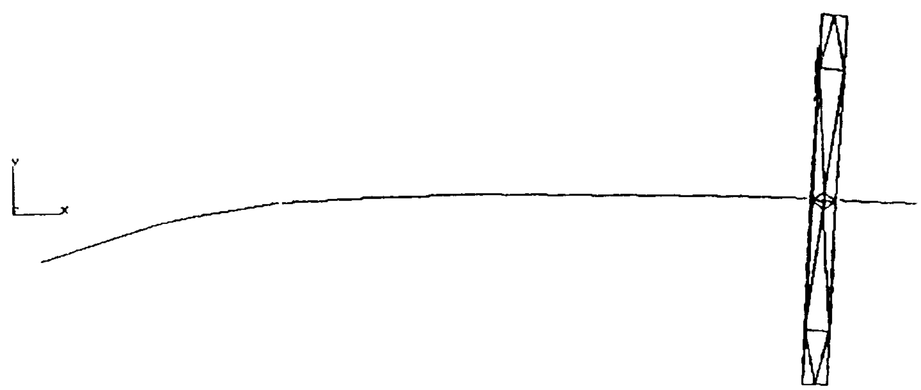

FIGURE 4.b: FLEXIBLE DISK MODEL - FIRST BENDING MODE
FREQUENCY = 462.2 Hz

FIGURE 4: HPOTP ROTOR MODELS - FIRST BENDING MODES

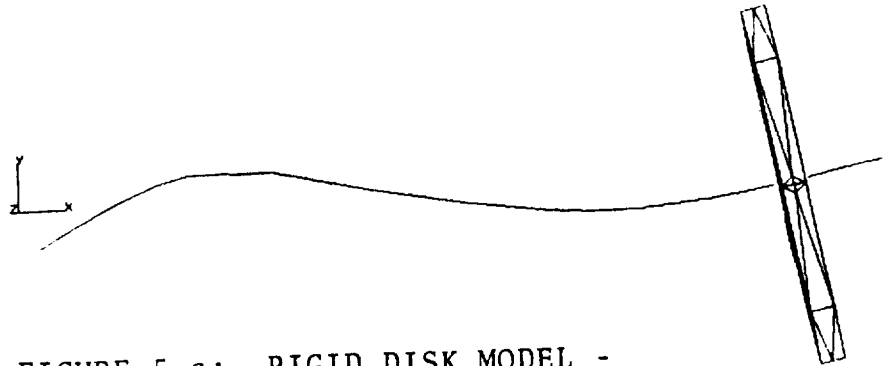

FIGURE 5.a: RIGID DISK MODEL -
SECOND BENDING MODE
FREQUENCY = 1072 Hz

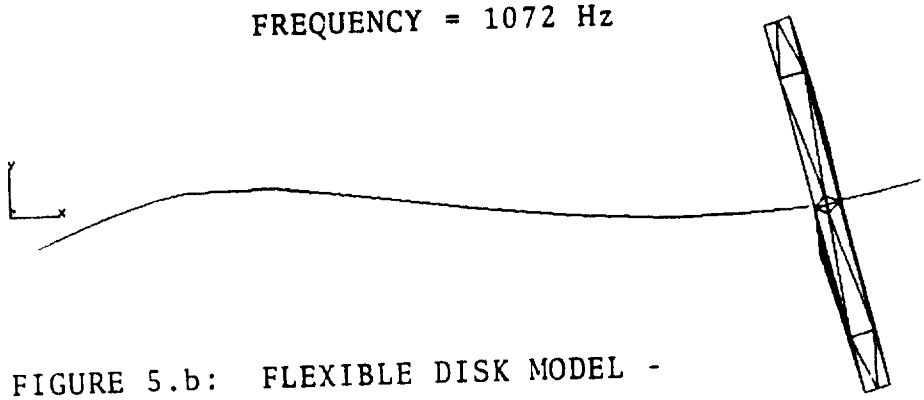

FIGURE 5.b: FLEXIBLE DISK MODEL -
SECOND BENDING MODE, DISK
IN-PHASE
FREQUENCY = 926.7 Hz

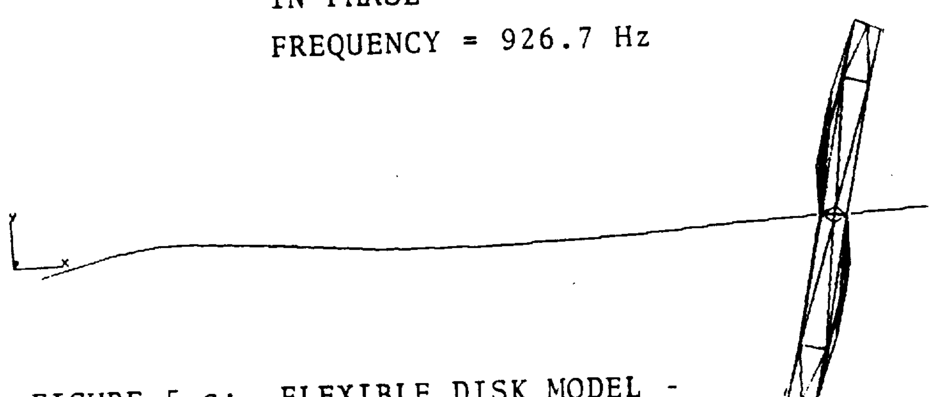

FIGURE 5.c: FLEXIBLE DISK MODEL -
SECOND BENDING MODE, DISK OUT-OF-PHASE
FREQUENCY = 1652.0 Hz

FIGURE 5: HPOTP ROTOR MODELS - SECOND BENDING MODES

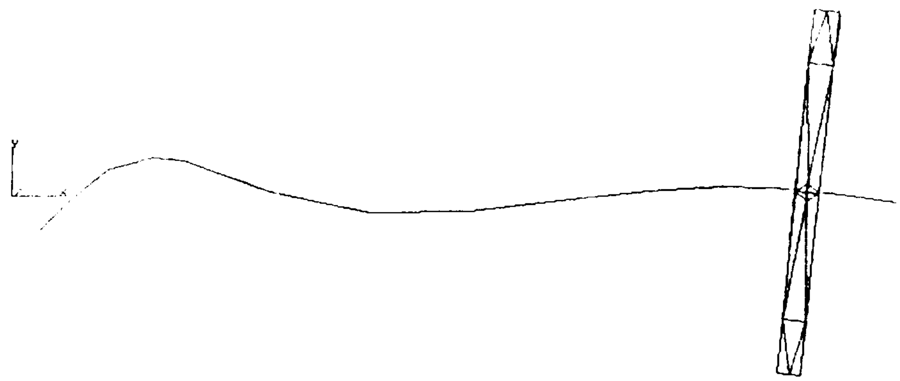

FIGURE 6.a: RIGID DISK MODEL - THIRD BENDING MODE
FREQUENCY = 1983 Hz

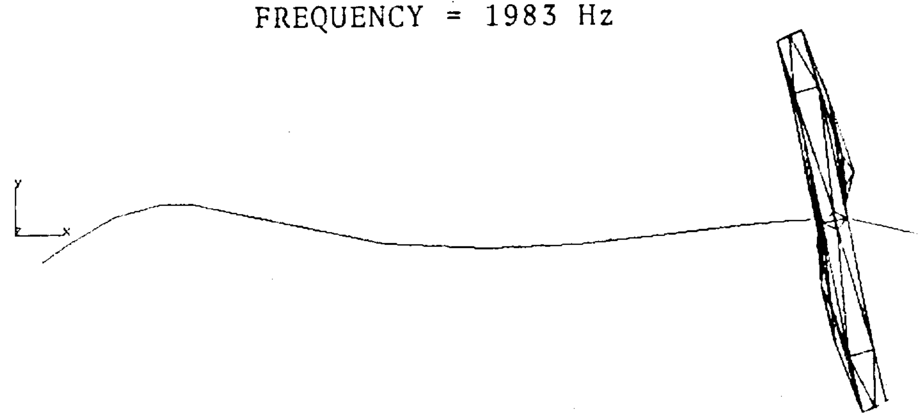

FIGURE 6.b: FLEXIBLE DISK MODEL - THIRD BENDING MODE
FREQUENCY = 1773 Hz

FIGURE 6: HPOTP ROTOR MODELS - THIRD BENDING MODES

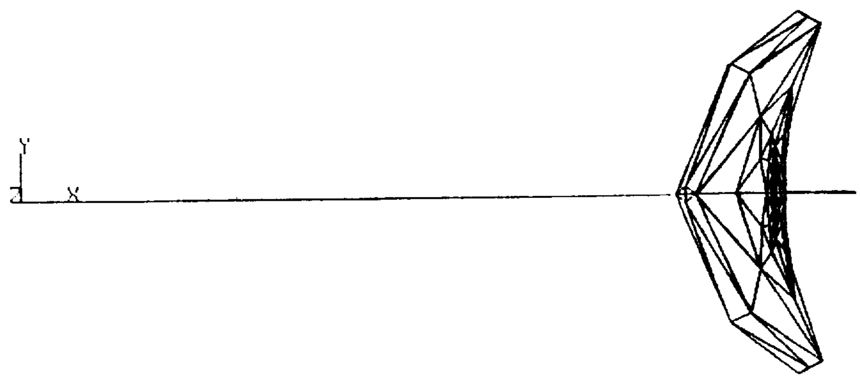

FIGURE 7: HPOTP FLEXIBLE DISK ROTOR MODEL - DISK MODE

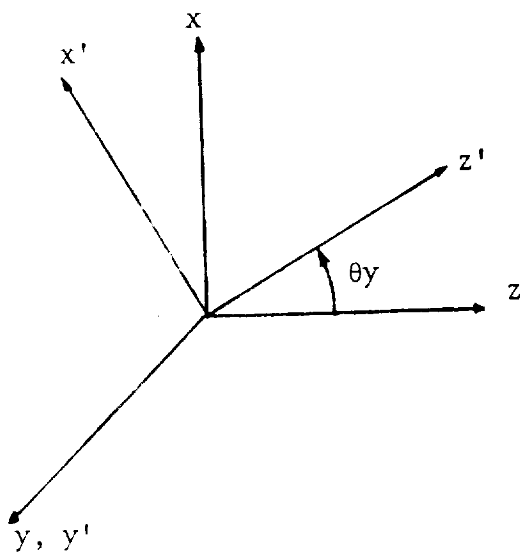

FIGURE 8.a

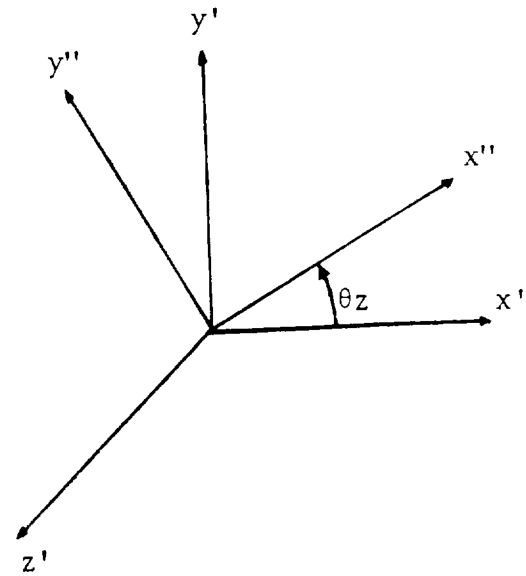

FIGURE 8.b

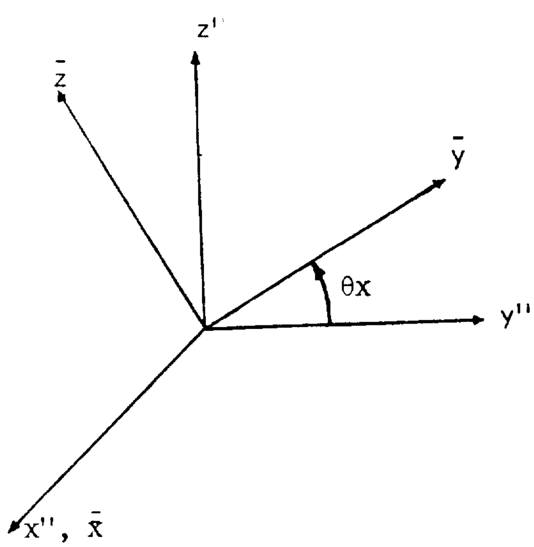

FIGURE 8.c

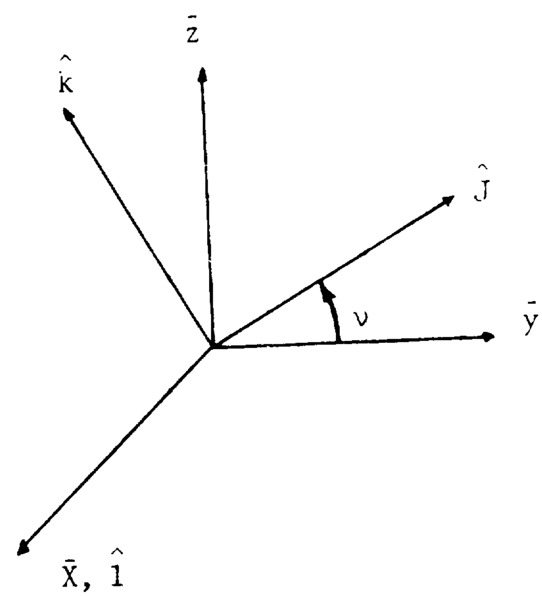

FIGURE 8.d

FIGURE 8: COORDINATE SYSTEM ROTATION SEQUENCE